



UNIVERSITY OF  
ILLINOIS LIBRARY  
AT URBANA-CHAMPAIGN  
BOOKSTACKS

Digitized by the Internet Archive  
in 2011 with funding from  
University of Illinois Urbana-Champaign

<http://www.archive.org/details/heterogeneousinf1220kwon>





# BEBR

FACULTY WORKING  
PAPER NO. 1220

THE LIBRARY OF THE  
APR 01 1985  
UNIVERSITY OF ILLINOIS  
URBANA-CHAMPAIGN

Heterogeneous Information, Market Efficiency  
and the Volatility of Equilibrium Stock Prices

*Young K. Kwon*  
*Hun Y. Park*



# **BEBR**

FACULTY WORKING PAPER NO 1220

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

January 1986

Heterogeneous Information, Market Efficiency  
and the Volatility of Equilibrium Stock Prices

Young K. Kwon, Professor  
Department of Accountancy

Hun Y. Park, Assistant Professor  
Department of Finance





# HETEROGENEOUS INFORMATION, MARKET EFFICIENCY AND THE VOLATILITY OF EQUILIBRIUM STOCK PRICES

## ABSTRACT

This paper investigates the impact of heterogeneous information among traders on market efficiency and the volatility of stock prices. After modeling the behavior of the traders in market equilibrium, this paper shows that 1) the volatility of equilibrium stock prices increases prior to disclosure of public information; 2) this phenomenon can be attributed to the speculative behavior of traders with heterogeneous information. More importantly, the paper also shows that this speculative behavior itself is the driving force for the efficient pricing mechanism. Thus, the increase in the volatility of stock prices prior to disclosure of public information is not inconsistent with the market efficiency hypothesis. These results also provide some important implications for event studies.



# HETEROGENEOUS INFORMATION, MARKET EFFICIENCY AND THE VOLATILITY OF EQUILIBRIUM STOCK PRICES

## 1. INTRODUCTION

Over the past two decades, a number of studies have examined the impact of various "public information" on stock prices and their volatilities to test market efficiency. Given the popularity of the event studies in the literature, it is remarkable that little attention has been paid to the underlying information adjustment process itself. As pointed out by Brown, Lockwood and Lummer [1], the characteristic common to the event studies is the attempt to quantify the influence of otherwise qualitative events.

The predominant methodology adopted in the event studies is the residual analysis based on capital asset pricing models (e.g., Sharpe [15] and its variations). In these studies, the coefficients of the return-generating model are assumed to remain constant for the predetermined time interval on either side of the event date. One of the event studies quoted quite often in the literature is the consideration of the effects of stock splits on share prices by Fama, Fisher, Jensen and Roll (FFJR) [4]. They found significant changes in the average variance of the residuals of the return-generating model (i.e., heteroscedasticity) during the months constituting the split period. The heteroscedasticity has been substantiated also by other studies in different contexts, questioning the validity of the return-generating model (see for example Hsu [10], Giaccotto and Ali [5] and Brown, Lockwood and Lummer [1]).

However, the previous studies to our knowledge have not investigated directly the underlying information adjustment process itself in relation to market efficiency and why the heteroscedasticity is so widespread, even though informally discussed in some study such as FFJR.

Relatedly, several studies (LeRoy and Porter [12], Shiller [16, 17], and Grossman and Shiller [7]), have examined the variability of stock prices in terms of the fundamental factors of the intrinsic value of common stocks to assess market efficiency.<sup>1</sup> The valuation model commonly used by financial economists asserts that corporation stock prices equal the present values of future cash flows (i.e., dividends) discounted by appropriate rates. LeRoy and Porter [12] and Shiller [16] have tested whether movements in stock prices can be explained by information about the numerator of the present value model (i.e., subsequent dividends) assuming a constant discount rate. The conclusion that these studies have in common is that stock prices are too volatile to accord with efficient markets, and thus the variability of stock prices cannot be accounted for simply by information regarding the future dividend payments. On the other hand, Grossman and Shiller [7] have considered whether the substantial volatility of stock prices can be attributed to information regarding discount interest rates. Using the marginal rate of substitution between consumption today and consumption in the future, they provide a convincing evidence that the unexpected high variability of stock prices cannot be explained by the discount factor either. In addition, Shiller [16] has argued also that "the failure of the efficient markets model is thus so dramatic that it would be impossible to attribute the failure to such things as data

errors, price index problems, or changes in tax laws." However, it is important to note that the "efficient market" models employed in the previous studies are on the fundamental assumption that informations in the market are homogeneous across all traders.<sup>2</sup>

The purpose of this paper is to analyze the behavior of traders who have heterogeneous information and to investigate its impact on market efficiency and the volatility of stock prices. In particular, this paper shows that 1) the volatility of equilibrium stock prices increases prior to disclosure of public information; 2) this phenomenon can be attributed to the speculative behavior of traders with heterogeneous information. More importantly, this paper also shows that the speculative behavior itself is the driving force for the efficient pricing mechanism. Thus the increase in the volatility of stock prices prior to disclosure of public information is not inconsistent with the concept of market efficiency. These results provide some important implications for the event studies. In the presence of a series of public information over time, this study also sheds a light on the long-time controversy why the variability of stock prices is so much dramatic.

This paper is organized as follows. Section 2 models, under some simple assumptions, the behavior of traders who have heterogenous information, and Section 3 derives some implications of the model regarding the volatility of stock prices and its relation with the efficient market hypothesis. Section 4 contains a brief summary.

## 2. EQUILIBRIUM MODEL WITH HETEROGENEOUS INFORMATION

Let us consider a simplified world in which stocks of a single firm are traded and there are large but equal number  $N$  of buyers and

sellers. Prior to public release of new information, investors will have the incentive to privately learn about the nature of forthcoming information. Unless private information is perfect across all investors, they will trade stocks based on their private (heterogeneous) information and thus new information may be reflected in stock prices prior to its public release. Following Hirshleifer [9], such markets will be called "speculative" markets. If new information is publicly released at discrete points in time, investors may speculate between consecutive time points of public information. However, as the time until public information approaches zero, investors' expectations will be more homogeneous and, therefore, speculative opportunities will disappear with disclosure of public information.

Denote buyers' and sellers' expected stock prices prior to disclosure of public information by  $\bar{v}$  and  $\underline{v}$ , respectively, where  $\bar{v} > \underline{v}$ . In addition, let us assume that buyers' and sellers' preferences are characterized by the following short-term trading profit functions, respectively:

$$(1) \quad \begin{aligned} u_1 &= \bar{v} - c \text{ for buyer} \\ u_2 &= d - \underline{v} \text{ for seller,} \end{aligned}$$

where  $c$  is the striking price plus the cost of searching the best striking price (for buyer) and  $d$  represents the striking prices less the cost of searching the best striking price (for seller).<sup>3</sup> For analytic convenience, each buyer and seller is assumed to trade exactly

one share of the stock if and only if his or her expected profit is non-negative (i.e.,  $Eu_1 \geq 0$  and  $Eu_2 \geq 0$ ). Buyers are indexed by per seller search cost  $s$ , where  $k(s) = 1/a(\bar{v} - \underline{v})$  is the probability density of  $s$  with support  $0 < s \leq a(\bar{v} - \underline{v})$  and  $a$  is a positive constant. On the other hand, sellers are indexed by per buyer search cost  $t$ , where  $l(t) = 1/a(\bar{v} - \underline{v})$  is the probability density of  $t$  with support  $0 < t \leq a(\bar{v} - \underline{v})$ . Thus,  $Nk(s)$  and  $Nl(t)$  represent the number of buyers of type  $s$  and the number of sellers of type  $t$ , respectively, and the search costs are smaller if and only if buyers' and sellers' expectations are more homogeneous. Note that buyers and sellers are identically distributed with respect to their search costs. Since buyers in one period may become sellers in another period and vice versa, the identical distribution assumption appears reasonable.

Assume that buyers (sellers) know the probability distribution of the stock prices  $x$  ( $y$ ) that sellers (buyers) are willing to sell (pay) for. Denote by  $F(x)$  and  $G(y)$  the probability distribution functions of  $x$  and  $y$  with support  $\underline{x} \leq x \leq \bar{x}$  and  $\underline{x} \leq y \leq \bar{x}$  for buyers and sellers, respectively.<sup>4</sup>

Following McCall [4], the optimal buver behavior can be described as follows. Consider a buyer of type  $s$ , and suppose that  $Q$  is the smallest of the  $x$  values that he/she has observed. If an additional seller is randomly sampled, the gross increase of his/her expected profit is

$$(2) \quad \phi(0) = \int_{\underline{x}}^0 (Q-x)dF(x) = \int_{\underline{x}}^0 F(x)dx.$$

Since the additional sample costs  $s$ , the buyer will search the market until  $x \leq Q(s)$ , where the critical value of  $Q(s)$  is determined by the marginality condition

$$(3) \quad s = \phi(Q(s)).$$

Since  $\phi(Q)$  is increasing:

$$(4) \quad \phi'(Q) = F(Q) > 0$$

for all  $Q > \underline{x}$ , buyers are likely to search more in the average if and only if their search costs are smaller.

Following the same logic, the optimal sequential search strategy of a seller of type  $t$  can be described as: continue searching if an  $x < R(t)$  is observed; and trade with the buyer if he/she is willing to pay an  $x \geq R(t)$ , where the threshold  $R(t)$  solves

$$(5) \quad t = \int_{R(t)}^{\bar{x}} [y - R(t)] dG(y)$$

for all  $0 < t \leq a(\bar{v} - \underline{v})$ . Define

$$(6) \quad \psi(R) = \int_R^{\bar{x}} [y - R] dG(y) = \int_R^{\bar{x}} [1 - G(y)] dy$$

for all  $\underline{x} \leq R \leq \bar{x}$ . Then, condition (5) can be written as

$$(7) \quad t = \psi(R(t))$$

for all  $t$ . Since  $\psi(R)$  is decreasing:



$$(8) \quad \Psi'(R) = -[1-G(R)] < 0$$

for all  $R < \bar{x}$ , sellers are likely to search more in the average if and only if their search costs are smaller.

Theorem 1. If a market equilibrium exists, and it is characterized by distribution functions  $F(x)$  and  $G(x)$  with support  $\underline{x} \leq x \leq \bar{x}$ , the following must then hold (see Appendix A for proof).

$$(a) \quad \underline{v} \leq \underline{x} < \bar{x} \leq \bar{v};$$

$$(b) \quad \int_{\underline{x}}^{\bar{x}} F(x) dx \leq a(\bar{v} - \underline{v});$$

$$(c) \quad \int_{\underline{x}}^{\bar{x}} [1 - G(x)] dx \leq a(\bar{v} - \underline{v});$$

$$(d) \quad \left\{ \int_{\underline{x}}^{\bar{x}} [1 - G(y)] dy \right\} \cdot F'(x) = 1 - G(x);$$

$$(e) \quad \left\{ \int_{\underline{x}}^{\bar{x}} F(y) dy \right\} \cdot G'(x) = F(x);$$

for all  $\underline{x} \leq x \leq \bar{x}$

Theorem 2. If the search cost parameter  $a$  is not too large ( $\pi a \leq 2$ ), then market equilibrium exists and is characterized by the following (see Appendix B for proof):

$$(a) \quad F(x) = \sin r(x - \underline{x});$$

$$(b) \quad G(x) = 1 - \cos r(x - \underline{x});$$

$$(c) \quad \bar{x} - \underline{x} = \frac{\pi}{2r};$$

for all  $\underline{x} \leq x \leq \bar{x}$ , where

$$(9) \quad \underline{v} \leq \underline{x} \leq \frac{1}{4-\pi} \cdot \text{Min}\{(2-\pi+\pi a)\underline{v} + (2-\pi a)\bar{v}, (4-\pi)[(1-a)\bar{v} + a\underline{v}]\}$$

and

$$(10) \quad (1-a)\underline{v} + a\bar{v} - \underline{x} \leq \frac{1}{r} \leq \frac{2}{\pi-2} \cdot \text{Min}\{(1-a)\bar{v} + a\underline{v} - \underline{x}, \frac{\pi-2}{2} a(\bar{v}-\underline{v})\}.$$

The following implications are immediate from Theorems 1 and 2:

i) The price dispersion ( $\bar{x} - \underline{x}$ ) is larger if either search costs are larger (i.e.,  $a$  is larger) or traders' expectations get more heterogeneous (i.e.,  $\bar{v} - \underline{v}$  gets larger); ii) The price dispersion is positively related to the search cost parameter ( $a(\bar{v} - \underline{v})$ ).

### 3. VOLATILITY AND MARKET EFFICIENCY

In the previous section, we have shown that equilibrium stock prices prior to disclosure of public information (let us call it the "speculative price" as opposed to the "normal price" subsequent to public information) critically depends on speculators' searching behavior due to heterogeneous information. In order to examine the volatility of speculative prices relative to the one of normal prices and its relation with market efficiency, let us consider an arbitrary but fixed period from time  $\tau = 0$  to time  $\tau = 1$ . For analytic tractability, we assume that the relevant state for the period is realized

at  $\tau = 0$ , but that the state information is to be made public at  $\tau = 1$ . Denote by  $v$  the equilibrium stock price that is to be realized subsequent to public information at  $\tau = 1$ . Consider the speculative market at time  $\tau$  ( $0 < \tau < 1$ ). Since the price  $v$  is not likely to be known to investors at the time  $\tau$ , some investors' estimates of  $v$  may be larger than others' unless private information is homogeneous. Denote by  $\underline{v}(\tau)$  and  $\bar{v}(\tau)$  speculative sellers' and buyers' estimates of  $v$  at time  $\tau < 1$ , respectively. The speculation market is then active at  $\tau$  if and only if  $\underline{v}(\tau) < \bar{v}(\tau)$ .

In order to avoid indeterminate striking prices for speculative trading, let us assume that speculative sellers are Stackelberg leaders: a seller of type  $t$  sets his/her selling price at the individual reservation price  $R(t)$  and waits until a buyer who is willing to pay the price arrives. Given this assumption, the speculative price or actual striking price  $x$  is distributed by  $F(x|\tau)$  at time  $\tau$ , and thus its mean can be written as

$$(11) \quad E(x|\tau) = \int_{\underline{x}(\tau)}^{\bar{x}(\tau)} x \, dF(x|\tau) = \underline{x}(\tau) + \frac{1}{r(\tau)} \left( \frac{\pi}{2} - 1 \right)$$

where the quantities  $\underline{x}(\tau)$  and  $r(\tau)$  satisfy (9) and (10).

Let us define

$$(12) \quad b(\tau) = E(x|\tau) - v \text{ and } z(\tau) = x(\tau) - E(x|\tau)$$

for  $0 < \tau < 1$ . The quantity  $b(\tau)$  measures the deviation of the average speculative price from the stock price subsequent to public

information. We shall call  $b(\tau)$  speculators' average bias. Depending upon the reliability of their private information, speculators may be biased positively ( $b(\tau) \geq 0$ ) or negatively ( $b(\tau) \leq 0$ ). On the other hand, the variable  $z(\tau)$  measures the deviation of an actual striking price from its mean. We shall call  $z(\tau)$  speculators' trading risk component. Note that the trading risk  $z(\tau)$  is "small" if and only if the price dispersion  $\bar{x}(\tau) - \underline{x}(\tau)$  of speculative prices  $x(\tau)$  is small.

Then, the speculative price  $x(\tau)$  at time  $\tau$  ( $0 < \tau < 1$ ) can be written as:

$$(13) \quad x(\tau) = v + b(\tau) + z(\tau).$$

Furthermore, the trading risk  $z(\tau)$  can be characterized by<sup>5</sup>

$$(14) \quad E[z(\tau)] = 0 \text{ and } \text{Var}[z(\tau)] = \frac{\pi - 3}{[r(\tau)]^2}$$

where

$$(15) \quad 0 < \frac{1}{r(\tau)} \leq a[\bar{v}(\tau) - \underline{v}(\tau)].$$

Equation (13) implies that speculators are exposed to three risks: the volatility of the normal price  $v$  (i.e., the intrinsic risk of the firm); the risk due to errors of heterogeneous information ( $b(\tau)$ ); and, the trading risk of speculators ( $z(\tau)$ ). Then it becomes clear that the speculative price  $x(\tau)$  must be more volatile than the normal price  $v$  unless the normal price  $v$ , the average bias  $b(\tau)$  and the trading risk  $z(\tau)$  are negatively correlated. Recall that the results are based on the assumption that speculative sellers are Stackelberg

leaders. However, we may assume alternatively that speculative buyers are Stackelberg leaders. Since the equilibrium speculative price must be distributed between the two distribution functions  $F(x)$  and  $G(x)$ , all of the results in this paper should be intact with only minor changes even when neither buyers nor sellers are Stackelberg leaders.

It can be easily shown also that the higher volatility of stock prices prior to disclosure of public information is not inconsistent with the efficient market hypothesis and the very speculative activities are driving forces of the efficient pricing mechanism as long as speculators are less biased as the time to subsequent information approaches zero:

$$(16) \quad \lim_{\tau \rightarrow 1} [\bar{v}(\tau) - v] = \lim_{\tau \rightarrow 1} [\underline{v}(\tau) - v] = 0$$

In order to see this, let us rewrite equation (13) as

$$[x(\tau) - v]^2 = [b(\tau)]^2 + 2b(\tau)z(\tau) + [z(\tau)]^2.$$

Then,

$$(17) \quad E[x(\tau) - v]^2 = [b(\tau)]^2 + \text{Var}[z(\tau)]$$

(cf. (14)). Let  $\varepsilon > 0$  be arbitrary but fixed. Using the Chebyshev inequality, we have

$$(18) \quad \text{Prob}[|x(\tau) - v|^2 \geq \varepsilon] \leq \frac{1}{\varepsilon} E[x(\tau) - v]^2.$$

On the other hand, it follows from (12) that

$$(19) \quad |b(\tau)| \leq \text{Max}[|\bar{v}(\tau) - v|, |\underline{v}(\tau) - v|].$$

Also, (14) and (15) yield

$$(20) \quad \text{Var}[z(\tau)] \leq K \cdot [|\bar{v}(\tau) - v| + |v - \underline{v}(\tau)|]^2$$

where

$$K = (\pi - 3)a^2.$$

Combining (17), (18), (19) and (20), we can obtain from (16):

$$(21) \quad \lim_{\tau \rightarrow 1} \text{Prob}[|x(\tau) - v|^2 \geq \varepsilon] = 0$$

Therefore, as long as speculators become less biased as the time to public information disclosure approaches zero, the speculative price  $x(\tau)$  must converge to the normal price  $v$  with probability one. This implies that the speculative behavior itself of investors with heterogeneous information leads to the efficient market and thus the high volatility of stock prices prior to public information disclosure is not inconsistent with the efficient market hypothesis. Conversely, speculation may take place if and only if the market is efficient. In other words, to the extent that investors' heterogeneous information is confirmed by subsequent disclosure of public information, market efficiency appears necessary for rational speculation to occur.

The implication of the model developed in this paper for the volatility of common stock prices may provide a direct explanation for the observed structural changes in the return-generating model of the event studies, suggesting the necessity of model refinements in testing market efficiency.

In addition, in the presence of infinite number of public information disclosure over time, this implication of speculative behavior due to heterogeneous information sheds a light on explaining the "substantial" volatility of stock prices.

#### 4. SUMMARY

This paper analyzes the behavior of traders who have heterogeneous information and investigates its impact on market efficiency and the volatility of stock prices. In particular, it is shown that 1) the volatility of equilibrium stock prices increases prior to disclosure of public information; 2) this phenomenon can be attributed to the speculative behavior of traders with heterogeneous information. More importantly, this paper also shows that the speculative behavior itself is the driving force for the efficient pricing mechanism. Thus, the increase in the volatility of stock prices prior to disclosure of public information is not inconsistent with the concept of market efficiency. These results provide some important implications for the event studies and the long-time controversial issue, the "substantial" volatility of stock prices.

FOOTNOTES

<sup>1</sup>The volatility measures have been used to assess the information processing efficiency in other markets: see for example Shiller [18] and Singleton [19] for bonds markets, and Huang [11] for foreign currency markets.

<sup>2</sup>Several studies (e.g., Grossman [6], and Grossman and Stiglitz [8]) have examined the competitive price system in different contexts, i.e., how the price system conveys information from informed investors to uninformed investors. However, it is important to note that inherent in those studies is the assumption that information among the informed investors is homogeneous.

<sup>3</sup>It is important to note that even though many trades are made through dealers or specialists in organized exchanges, its magnitude is small. For example, actual transactions with specialists (as opposed to investors' trades among themselves) account for only 12 percent of the transactions in the New York Stock Exchange.

<sup>4</sup>Since buyers and sellers are to be matched on a one-to-one basis, both  $x$  and  $y$  values must have the same support.

<sup>5</sup>From (12),  $x(\tau) = v + [E(x|\tau) - v] + [x(\tau) - E(x|\tau)]$ . Thus (13) is obvious. In addition,  $\text{Var}[z(\tau)] = \text{Var}[x(\tau)] = \int_{\underline{x}}^{\bar{x}} x^2 dF(x|\tau) - E(x|\tau)^2$ , where  $F(x|\tau) = \sin[r(\tau)(x - \underline{x}(\tau))]$  for  $\underline{x}(\tau) \leq x \leq \bar{x}(\tau)$  from Theorem 2, and  $E(x|\tau)$  is given by (11). Therefore,  $\text{Var}[z(\tau)] = (\pi-3)/[r(\tau)]^2$  can be easily derived.



REFERENCES

- [1] Brown, K. C., Lockwood, L. J. and S. L. Lummer: "An Examination of Event Dependency and Structural Change in Security Pricing Models." Journal of Financial and Quantitative Analysis, 20, (September 1985), 315-334.
- [2] Fama, E.: "The Behavior of Stock Market Prices," Journal of Business 38 (1965), 34-105.
- [3] \_\_\_\_\_: "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance 25 (1970), 383-420.
- [4] \_\_\_\_\_, L. Fisher, M. Jensen and R. Roll: "The Adjustment of Stock Prices to New Information," International Economic Review 10 (1969), 1-21.
- [5] Giaccotto, C. and M. Ali: "Optimal Distribution-Free Tests Further Guidance of Heteroscedasticity in the Market Model," Journal of Finance 37 (December 1982), 1247-1258.
- [6] Grossman, S.: "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information," Journal of Finance (1976), 573-585.
- [7] \_\_\_\_\_ and R. Shiller: "The Determinates of the Variability of Stock Market Prices," American Economic Review 71 (1981), 222-227.
- [8] Grossman, S. and J. Stiglitz: "Information and Competitive Price System," American Economic Review 66 (1976), 246-253.
- [9] Hirshleifer, J.: "The Private and Social Value of Information and the Reward to Inventive Activity," American Economic Review 61 (1971), 561-574.

- [10] Hsu, D. A.: "Test for Variance Shift to an Unknown Time Point." Journal of the Royal Statistical Society 26 (1977), 279-284.
- [11] Huang, R. D.: "Efficiency and the Volatility of Floating Exchange Rates," Journal of Finance (1981).
- [12] LeRoy, S. and R. Porter: "The Present Value Relation: Tests Based on Implied Variance Bounds," Econometrica 49 (1981), 555-574.
- [13] MacMann, R.: "Search and Market Equilibrium," Journal of Political Economy 88 (1980), 308-327.
- [14] McCall, J.: "Economics of Information and Job Search," Quarterly Journal of Economics 84 (1970), 113-126.
- [15] Sharpe, W. F.: "A Simplified Model for Portfolio Analysis," Management Science 9 (1963), 277-293.
- [16] Shiller, R.: "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" American Economic Review 71 (1981), 421-436.
- [17] \_\_\_\_\_: "The Use of Volatility Measures in Assessing Market Efficiency," Journal of Finance 36 (1981), 291-304.
- [18] \_\_\_\_\_: "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure," Journal of Political Economy 87 (1979), 1190-1219.
- [19] Singleton, K: "Expectations Models for the Term Structure and Implied Variance Bounds," Journal of Political Economy (1980).
- [20] Stigler, G.: "The Economics of Information," Journal of Political Economy 69 (1961), 213-225.

## Appendix A

### Proof of Theorem 1

Result (a) is obvious since buyers' and sellers' expected trading profits must be non-negative. Turning to result (b), assume that it does not hold:

$$(A1) \quad a(\bar{v} - \underline{v}) < \phi(\bar{x}) = \int_{\underline{x}}^{\bar{x}} F(x)dx.$$

Since  $\phi(Q)$  is strictly increasing by (4), this implies

$$(A2) \quad Q[a(\bar{v} - \underline{v})] < \bar{x}$$

(apply the strictly increasing inverse  $\phi^{-1}(\cdot)$  of  $\phi(\cdot)$  to (A1) and then use (3)). It then follows from (A2) that no buyers are going to pay the prices  $x$  with  $Q[a(\bar{v} - \underline{v})] < x \leq \bar{x}$ : a contradiction to the fact that  $\underline{x} \leq x \leq \bar{x}$  is the support of  $G(x)$ . In other words, condition (b) must hold in equilibrium. The proof of result (c) is similar.

Now we consider result (d). Let  $x$  be fixed such that  $\underline{x} < x \leq \bar{x}$ .  $F(x)$  is generated by the set of the reservation prices  $R(t)$  of sellers of type  $t$  with  $R(t) < x$  conditional upon the constraint that  $\underline{x} \leq R(t) \leq \bar{x}$ :

$$F(x) = \text{Prob} \{ R(t) < x | \underline{x} \leq R(t) \leq \bar{x} \}.$$

Since  $R(t) < x$  if and only if  $t > \psi(x)$  by (7) and (8), we have

$$F(x) = \frac{1}{\{\psi(\underline{x})/[a(\bar{v} - \underline{v})]\}} \int_{\psi(x)}^{\psi(\underline{x})} \ell(t)dt = 1 - \frac{\psi(x)}{\psi(\underline{x})}$$

for all  $\underline{x} < x \leq \bar{x}$ . Differentiating the above with respect to  $x$  and then using (6) and (8), we can easily obtain (d). Since the proof of result (e) is similar, this completes the proof of Theorem 1.

## Appendix B

### Proof of Theorem 2

Consider the system of differential equations:

$$(B1) \quad A F'(x) = 1 - G(x) \text{ and } B G'(x) = F(x)$$

where

$$A = \int_{\underline{x}}^{\bar{x}} [1 - G(y)] dy \text{ and } B = \int_{\underline{x}}^{\bar{x}} F(y) dy$$

(cf. (d) and (e) in Theorem 1). Eliminating  $F(x)$  from (B1), we have

$$(B2) \quad AB G''(x) + G(x) = 1.$$

Setting

$$(B3) \quad r = \frac{1}{\sqrt{AB}},$$

we can write the solution of (B2) in the form:

$$G(x) = 1 - c_1 \cos(rx - c_2)$$

for all  $\underline{x} \leq x \leq \bar{x}$ , where  $c_1$  and  $c_2$  are integration constants. It then follows from (B1) that  $F(x)$  must have the form:

$$F(x) = Brc_1 \sin(rx - c_2)$$

for all  $\underline{x} \leq x \leq \bar{x}$ . Since  $F(\bar{x}) = G(\bar{x}) = 1$ , we must then have

$$(B4) \quad Br = 1; \quad r\bar{x} - c_2 = n\pi + \frac{\pi}{2}; \quad \text{and } c_1 = (-1)^n$$

where  $n$  is an integer. Since  $Br = 1$ , (B3) yields

$$(B5) \quad A = B = \frac{1}{r}.$$

Since  $F(\underline{x}) = G(\underline{x}) = 0$ , we also have

$$(B6) \quad r\underline{x} - c_2 = m\pi; \quad \text{and } n - m = \text{an even integer.}$$

Since  $F(x) \geq 0$  and  $G(x) \leq 1$  for all  $\underline{x} \leq x \leq \bar{x}$ , however, the quantities  $(r\bar{x} - c_2)$  and  $(r\underline{x} - c_2)$  cannot differ by more than  $\frac{\pi}{2}$ . Thus,  $n - m = 0$

and (B6) can be rewritten as

$$(B6)' \quad r\underline{x} - c_2 = n\pi.$$

As a result, we can obtain

$$rx - c_2 = r(x - \underline{x}) + (r\underline{x} - c_2) = r(x - \underline{x}) + n\pi$$

and results (a)-(c) thus follow.

Turning to results (9) and (10), note that

$$(B7) \quad 0 < \frac{1}{r} \leq (\bar{v} - \underline{v}) \cdot \text{Min}[a, \frac{2}{\pi}]$$

(cf. (a) and (b) in Theorem 1 and result  $\bar{x} - \underline{x} = \frac{\pi}{2r}$ ). Also, observe that buyers of type  $s$  with  $\frac{1}{r} < s \leq a(\bar{v} - \underline{v})$  and sellers of type  $t$  with  $\frac{1}{r} < t \leq a(\bar{v} - \underline{v})$  search the market only once since

$$\int_{\underline{x}}^{\bar{x}} x dF(x) = \bar{x} - \frac{1}{r}, \quad \text{and} \quad \int_{\underline{x}}^{\bar{x}} y dG(y) = \underline{x} + \frac{1}{r}.$$

In view of the facts that

$$Eu_1(s) = \bar{v} - s - \int_{\underline{x}}^{\bar{x}} x dF(x) = \bar{v} - s - \underline{x} - \left(\frac{\pi}{2} - 1\right) \frac{1}{r}$$

for all  $\frac{1}{r} < s \leq a(\bar{v} - \underline{v})$ , and that  $Eu_1(s) \geq 0$ , we must have

$$(B8) \quad \left(\frac{\pi}{2} - 1\right) \frac{1}{r} \leq \bar{v} - a(\bar{v} - \underline{v}) - \underline{x} = (1-a)\bar{v} + a\underline{v} - \underline{x}.$$

Similarly, we obtain from  $Eu_2(t) \geq 0$  for  $t = a(\bar{v} - \underline{v})$  that

$$(B9) \quad \frac{1}{r} \geq a\bar{v} + (1-a)\underline{v} - \underline{x}.$$

In other words, we must determine endogenously the quantities  $r$  and  $\underline{x}$  such that conditions (B7), (B8) and (B9) are satisfied. While tedious, it is not difficult to show that results (9) and (10) follow from (B7), (B8) and (B9). The proof of Theorem 2 is herewith completed.

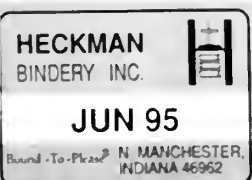












UNIVERSITY OF ILLINOIS-URBANA



3 0112 038252612